

# Accelerated expansion of the Universe without an inflaton and resolution of the initial singularity from Group Field Theory condensates

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## Abstract

We study the expansion of the Universe using an effective Friedmann equation obtained from the dynamics of GFT (Group Field Theory) isotropic condensates. The evolution equations are classical, with quantum correction terms to the Friedmann equation given in the form of effective fluids coupled to the emergent classical background. The occurrence of a bounce, which resolves the initial spacetime singularity, is shown to be a general property of the model. A promising feature of this model is the occurrence of an era of accelerated expansion, without the need to introduce an inflaton field with an appropriately chosen potential. We discuss possible viability issues of this scenario as an alternative to inflation.

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## 1. Introduction

Inflation, despite its undoubted success in explaining cosmological data and the numerous models studied in the literature, still remains a paradigm in search of a theory. The inflationary era should have occurred at the very early stages of our Universe, however the inflationary dynamics are commonly

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studied in the context of Einstein’s classical gravity and assuming the existence of a classical scalar field with a particularly tuned potential. Clearly, the onset of inflation [1, 2] and the inflationary dynamics must be addressed within a quantum gravity proposal. In this letter, employing results from Group Field Theory [3, 4] (GFT), we attempt to bridge the gap between the quantum gravity era and the standard classical cosmological model. In particular, in the context of GFT we propose a model that can account for an early accelerated expansion of our Universe in the absence of an inflaton field. We hence show that modifications in the gravitational sector of the theory can account for its early stage dynamics. Indeed, it is reasonable to expect that quantum gravity corrections at very early times – when geometry, space and time lose the meaning we are familiar with – may effectively lead to the same dynamics as the introduction of a hypothetical inflaton field with a suitable potential to satisfy cosmological data.

Group Field Theory is a non-perturbative and background independent approach to quantum gravity. In GFT, the fundamental degrees of freedom of quantum space are associated to graphs labelled by algebraic data of group theoretic nature. The quantum spacetime is seen as a superposition of discrete quantum spaces, each one generated through an interaction of fundamental building blocks (called “quanta of geometry”), typically considered as tetrahedra. In the continuum classical limit, one then expects to recover the standard dynamics of General Relativity. In this sense, the notion of space-time geometry, gravity and time can be seen as emergent phenomena. Group Field Theory cosmology is built upon the existence of a condensate state of GFT quanta, interpreted macroscopically as a homogeneous universe.

## 2. GFT Cosmology

In this work we study the properties of solutions of the modified Friedmann equation [5], obtained within the context of GFT condensates. The condensate wave function can be written as  $\sigma_j = \rho_j e^{i\theta_j}$ , where  $j$  is a representation index. Evolution is purely relational, thus all dynamical quantities are regarded as functions of a massless scalar field  $\phi$ . Derivatives with respect to  $\phi$  will be denoted by a prime. There is a conserved charge associated to  $\theta_j$ :

$$\rho_j^2 \theta'_j = Q_j. \tag{1}$$

The modulus satisfies the equation of motion

$$\rho_j'' - \frac{Q^2}{\rho_j^3} - m_j^2 \rho_j = 0, \quad (2)$$

leading to another conserved current, the *GFT energy*:

$$E_j = (\rho_j')^2 + \frac{Q_j^2}{\rho_j^2} - m_j^2 \rho_j^2, \quad (3)$$

where  $m_j^2$  can be expressed in terms of coefficients in the corresponding GFT theory, see Ref. [5] for details. Equation (2) admits the following solution

$$\rho_j(\phi) = \frac{e^{(-b-\phi)\sqrt{m_j^2}} \Delta(\phi)}{2\sqrt{m_j^2}}, \quad (4)$$

where

$$\Delta(\phi) = \sqrt{a^2 - 2ae^{2(b+\phi)\sqrt{m_j^2}} + e^{4(b+\phi)\sqrt{m_j^2}} + 4m_j^2 Q_j^2} \quad (5)$$

and  $a, b$  are integration constants. From Eq. (3) follows

$$E_j = a, \quad (6)$$

whereas the charge  $Q_j$  contributes to the canonical momentum of the scalar field (see Ref. [5])

$$\sum_j Q_j = \pi_\phi. \quad (7)$$

The dynamics of macroscopic observables is defined through that of the expectation values of the corresponding quantum operators. In GFT, as in Loop Quantum Gravity, the fundamental observables are geometric operators, such as areas and volumes. The volume of space at a given value of relational time  $\phi$ , is thus obtained from the condensate wave function as

$$V = \sum_j V_j \rho_j^2, \quad (8)$$

where  $V_j \propto j^{3/2} \ell_{Pl}$  is the eigenvalue of the volume operator corresponding to a given representation  $j$ . Using this as a definition and differentiating w.r.t. relational time  $\phi$  one obtains, as in Ref. [5] the following equations, which

play the rôle of effective Friedmann (and acceleration) equations describing the dynamics of the cosmos as it arises from that of a condensate of spacetime quanta

$$\frac{V'}{V} = \frac{2 \sum_j V_j \rho_j \rho'_j}{\sum_j V_j \rho_j^2}, \quad (9)$$

$$\frac{V''}{V} = \frac{2 \sum_j V_j (E_j + 2m_j^2 \rho_j^2)}{\sum_j V_j \rho_j^2}. \quad (10)$$

In the context of GFT, spacetime is thus seen to emerge in the hydrodynamic limit of the theory; the evolution of a homogeneous and isotropic Universe is completely determined by that of its volume. Notice that the above equations are written in terms of functions of  $\phi$ . In fact, as implied by the background independence of GFT, and more in general of any theory of quantum geometry, *a priori* there is no spacetime at the level of the microscopic theory and therefore no way of selecting a coordinate time. Nevertheless, we will show how it is possible to introduce a preferred choice of time, namely proper time, in order to study the dynamics of the model in a way similar to the one followed for standard homogeneous and isotropic models. This will be particularly useful for the study of the accelerated expansion of the Universe. In the following we will restrict our attention to the case in which the condensate belongs to one particular representation of the symmetry group. This special case can be obtained from the equations written above by considering a condensate wave function  $\sigma_j$  with support only on  $j = j_0$ . Representation indices will hereafter be omitted. Hence, we have

$$\frac{V'}{V} = 2 \frac{\rho'}{\rho} \equiv 2g(\phi), \quad (11)$$

$$\frac{V''}{V} = 2 \left( \frac{E}{\rho^2} + 2m^2 \right). \quad (12)$$

As  $\phi \rightarrow \pm\infty$ ,  $g(\phi) \rightarrow \sqrt{m^2}$  and the standard Friedmann and acceleration equations with a constant gravitational coupling and a fluid with a stiff equation of state are recovered. We will introduce proper time by means of the relation between velocity and momentum of the scalar field

$$\pi_\phi = \dot{\phi} V. \quad (13)$$

Furthermore, we can *define* the scale factor as the cubic root of the volume

$$a \propto V^{1/3}. \quad (14)$$

We can therefore write the evolution equation of the Universe obtained from GFT in the form of an *effective Friedmann equation* ( $H = \frac{\dot{V}}{3V}$  is the Hubble expansion rate and  $\varepsilon = \frac{\dot{\phi}^2}{2}$  the energy density)

$$H^2 = \left( \frac{V'}{3V} \right)^2 \dot{\phi}^2 = \frac{8}{9} g^2 \varepsilon. \quad (15)$$

Using Eqs. (3), (7), (13) we can recast Eq. (15) in the following form

$$H^2 = \frac{8}{9} Q^2 \left( \frac{\gamma_m}{V^2} + \frac{\gamma_E}{V^3} + \frac{\gamma_Q}{V^4} \right), \quad (16)$$

where we introduced the quantities

$$\gamma_m = \frac{m^2}{2}, \quad \gamma_E = \frac{V_j E}{2}, \quad \gamma_Q = -\frac{V_j^2 Q^2}{2}. \quad (17)$$

The first term in Eq. (16) is, up to a constant factor, the energy density of a massless scalar field on a conventional FLRW background, whereas the others represent the contribution of effective fluids with distinct equations of state and express departures from the ordinary Friedmann dynamics. Respectively, the equations of state of the terms in Eq. (17) are given by  $w = 1, 2, 3$ , consistently with the (quantum corrected) Raychaudhuri equation Eq. (27). Effective fluids have been already considered in the context of LQC as a way to encode quantum corrections, see *e.g.* [6].

This equation reduces to the conventional Friedmann equation in the large  $\phi$  limit, where the contributions of the extra fluid components are negligible

$$H^2 = \frac{8\pi G}{3} \varepsilon. \quad (18)$$

Thus, consistency in the limit demands  $m^2 = 3\pi G$ , which puts some constraints on the parameters of the microscopic model based on its macroscopic limit (see Ref. [5]).

The interpretation of our model is made clear by Eq. (16). In fact the dynamics has the usual Friedmann form with a classical background represented by the scale factor  $a$  and quantum geometrical corrections given by two effective fluids, corresponding to the two conserved quantities  $Q$  and  $E$ . In the following we will consider for convenience Eq. (15) in order to study the properties of solutions.

Let us discuss in more detail the properties of the model at finite (relational) times. Eq. (11) predicts a bounce when  $g(\phi)$  vanishes. We denote by  $\Phi$  the “instant” when the bounce takes place. One can therefore eliminate the integration constant  $b$  in favour of  $\Phi$

$$b = \frac{\log\left(\sqrt{E^2 + 4m^2Q^2}\right)}{2\sqrt{m^2}} - \Phi. \quad (19)$$

We define the *effective* gravitational constant as

$$G_{\text{eff}} = \frac{1}{3\pi} g^2, \quad (20)$$

which can be expressed, using Eqs. (4), (11) as

$$G_{\text{eff}} = \frac{G(E^2 + 12\pi GQ^2) \sinh^2\left(2\sqrt{3\pi G}(\phi - \Phi)\right)}{\left(E - \sqrt{E^2 + 12\pi GQ^2} \cosh\left(2\sqrt{3\pi G}(\phi - \Phi)\right)\right)^2}. \quad (21)$$

Its profile is given in Figs. 1,2, in the cases  $E < 0$ ,  $E > 0$  respectively. Notice that it is symmetric about the line  $\phi = \Phi$ , corresponding to the bounce.

The energy density has a maximum at the bounce, where the volume reaches its minimum value

$$\varepsilon_{\text{max}} = \frac{1}{2} \frac{Q^2}{V_{\text{bounce}}^2}, \quad (22)$$

where

$$V_{\text{bounce}} = \frac{V_{j_0} \left(\sqrt{E^2 + 12\pi GQ^2} - E\right)}{6\pi G}. \quad (23)$$

Clearly, the singularity is always avoided for  $E < 0$  and, provided  $Q \neq 0$ , it is also avoided in the case  $E > 0$ . Moreover, if the GFT energy is negative, the energy density has a vanishing limit at the bounce for vanishing  $Q$ :

$$\lim_{Q \rightarrow 0} \varepsilon_{\text{max}} = 0, \quad E < 0. \quad (24)$$

Therefore in this limiting case the energy density is zero at all times. Nevertheless, the Universe will still expand following the evolution equations (11) and

$$\lim_{Q \rightarrow 0} V(\phi) = \frac{|E|V_{j_0} \cosh^2\left(\sqrt{3\pi G}(\phi - \Phi)\right)}{3\pi G}, \quad E < 0. \quad (25)$$

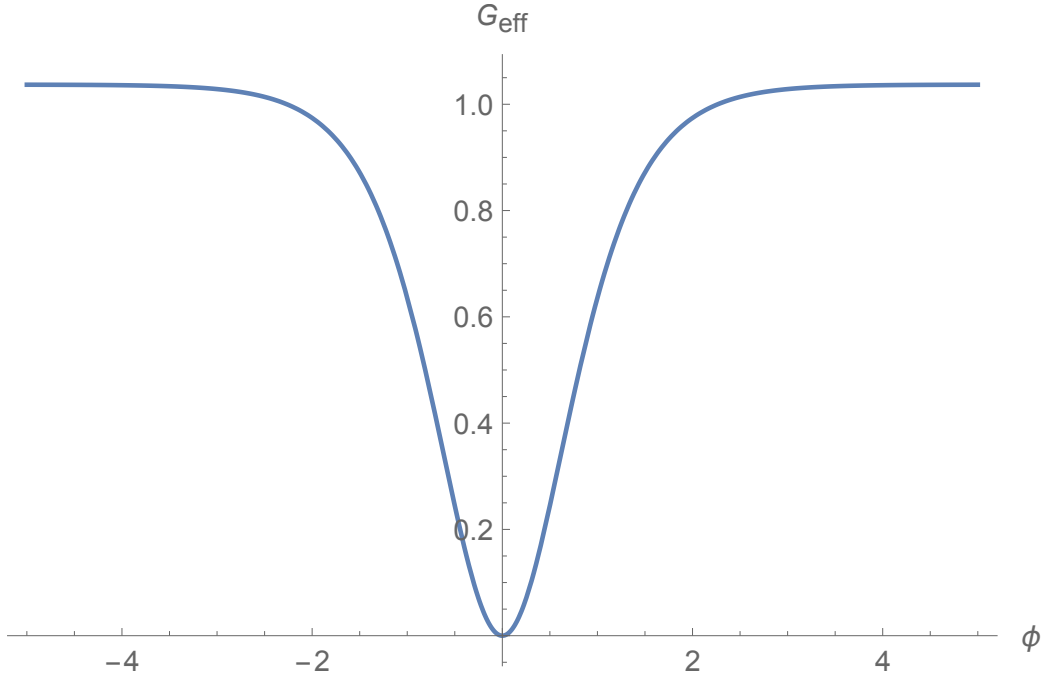


Figure 1:

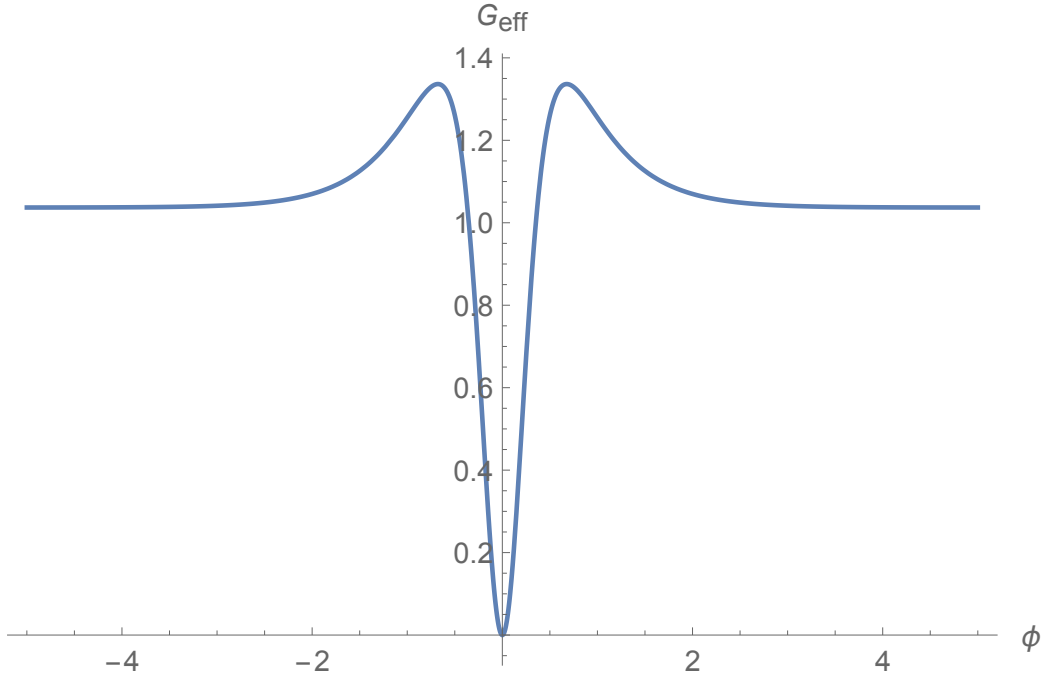


Figure 2: The effective gravitational constant as a function of relational time  $\phi$  for  $E < 0$  (Fig. 1) and  $E > 0$  (Fig. 2), in arbitrary units. There is a bounce replacing the classical singularity in both cases. The origin in the plots corresponds to the bounce, occurring at  $\phi = \Phi$ . The asymptotic value for large  $\phi$  is the same in both cases and coincides with Newton's constant. In the case  $E < 0$  this limit is also a supremum, whereas in the  $E > 0$  case  $G_{eff}$  has two maxima, equally distant from the bounce, and approaches Newton's constant from above.

This is to be contrasted with classical cosmology (18), where the rate of expansion is zero when the energy density vanishes.

It is possible to express the condition that the Universe has a positive acceleration in purely relational terms. In fact this very notion relies on the choice of a particular time parameter, namely proper time, for its definition. Introducing the scale factor and proper time as in Eqs. (13), (14) one finds

$$\frac{\ddot{a}}{a} = \frac{2}{3}\varepsilon \left[ \frac{V''}{V} - \frac{5}{3} \left( \frac{V'}{V} \right)^2 \right]. \quad (26)$$

We observe that the last equation can also be rewritten as

$$\frac{\ddot{a}}{a} = -\frac{4}{9}Q^2 \left( 4\frac{\gamma_m}{V^2} + 7\frac{\gamma_E}{V^3} + 10\frac{\gamma_Q}{V^4} \right). \quad (27)$$

We can trade the condition  $\ddot{a} > 0$  for having an accelerated expansion with the following one, which only makes reference to relational evolution of observables.

$$\frac{V''}{V} > \frac{5}{3} \left( \frac{V'}{V} \right)^2 \quad (28)$$

The two conditions are obviously equivalent. However, the second one has a wider range of applicability, since it is physically meaningful also when the scalar field has vanishing momentum. Making use of Eq. (11) the condition above can be rewritten as

$$4m^2 + \frac{2E}{\rho^2} > \frac{20}{3}g^2. \quad (29)$$

This is satisfied trivially in a neighbourhood of the bounce since  $g$  vanishes there and the l.h.s. of the inequality is strictly positive, see Figs. 3, 4. It is instead violated at infinity, consistently with a decelerating Universe in the classical regime.

### 3. Discussion

The dynamics of the Universe predicted by the GFT model is purely relational, *i.e.*, using the language of Ref. [7], it is expressed by the functional relation between *partial observables*, here given by the volume  $V$  and the scalar field  $\phi$ . According to this interpretation, physically meaningful statements about the predicted value of  $V$  can only be made in conjunction with



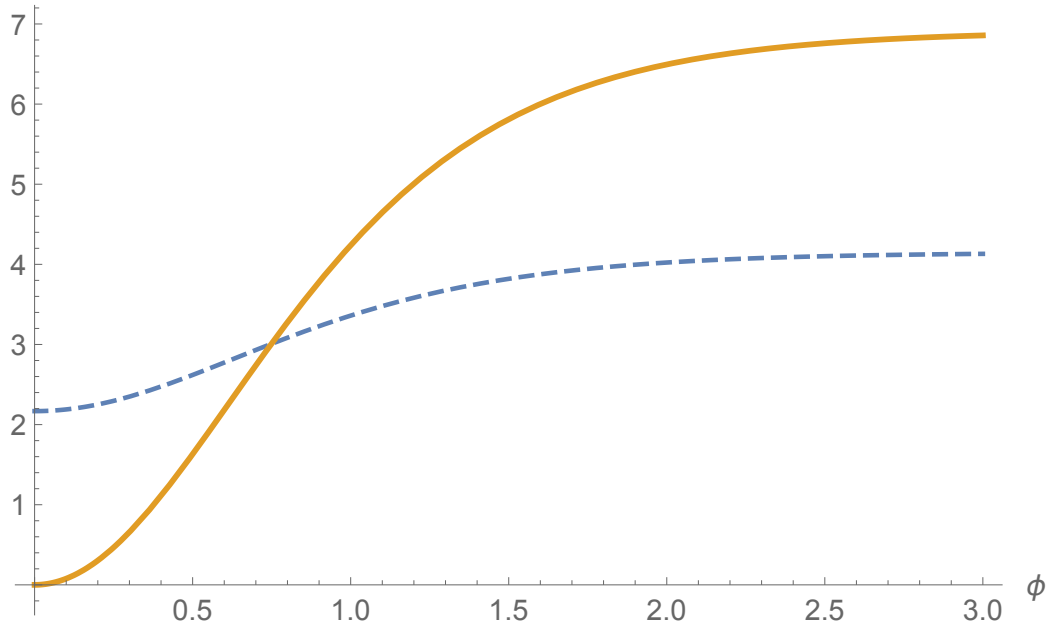


Figure 3:

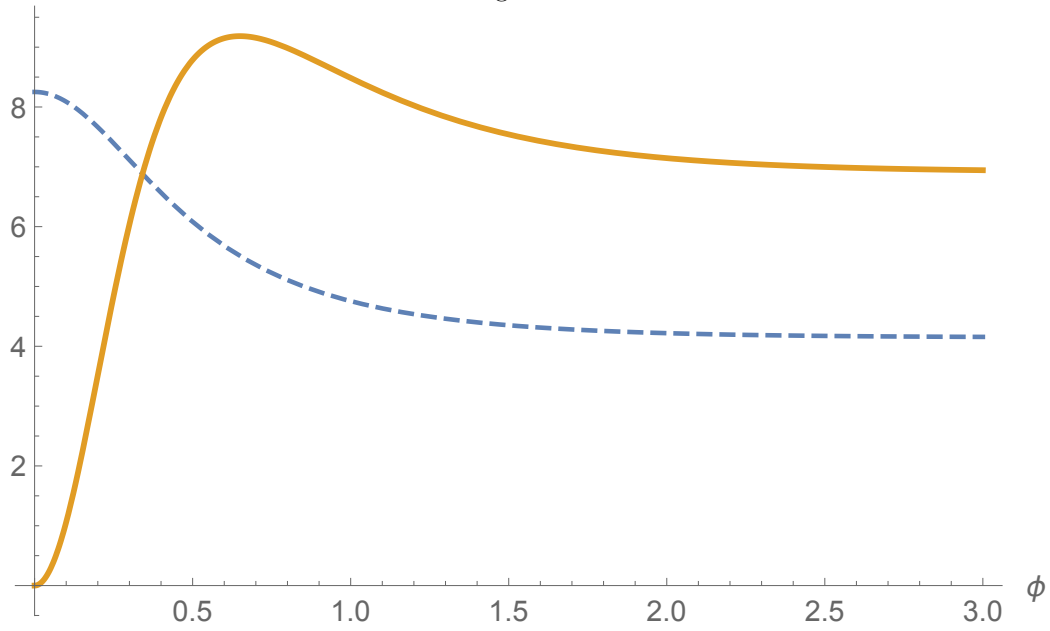


Figure 4: The l.h.s. and the r.h.s of inequality (29) as functions of the relational time  $\phi$  correspond to the dashed (blue) and thick (orange) curve respectively, in arbitrary units. When the dashed curve is above the thick one the Universe is undergoing an epoch of accelerated expansion following the bounce. Figure 3 corresponds to the case  $E < 0$ , whereas Fig. 4 is relative to the opposite case  $E > 0$ . Notice that for the latter there is a stage of maximal deceleration after exiting the “inflationary” era. After that the acceleration takes less negative values until it relaxes to its asymptotic value. For  $E < 0$  instead the asymptote is approached from below.

statements about the predicted outcome of a measurement of  $\phi$ . In fact, this interpretation is inspired by one of the main insights of GR, namely by the observation that coordinate time is purely gauge-dependent, and is therefore deprived of any physical meaning. Thus, it cannot be expected to play any rôle in the quantum theory either. In other words, the dynamics is entirely given by the so called *complete observables*, which in this model are exhausted by the functional relation  $V(\phi)$ . In a theory with gauge invariance, such quantities are the only ones having physical meaning; they can be seen as functions on the space of solutions modulo all gauges [7]. Gauge invariance of  $V(\phi)$  is trivially verified in classical cosmology; it is also valid at the quantum level, since gauge invariance of the volume operator follows from its general definition in GFT [4]. A discussion on the implementation of diffeomorphism invariance in GFT can be found in Ref. [8].

The reader might be interested in finding a closer correspondence between our discussion of relational dynamics and examples considered, *e.g.*, in Ref. [9]. In that work relational dynamics was obtained, both at the classical and the quantum level, adopting a canonical formulation and constructing *complete observables* in the case of simple models. However, we must point out that the definition of *partial* and *complete observables* is much more general and does not rely on a phase space structure, but only on the possibility of identifying gauge equivalence classes in the space of solutions<sup>1</sup>. Furthermore, since the quantum theory is not based on canonical quantization, it is not clear how a presymplectic structure might emerge in the classical limit from the full theory. It should nevertheless be possible to find such a geometric structure and a Hamiltonian at least for the cosmological sector of the theory considered here; an investigation which however lies beyond the scope of the present work.

Our classical model is free of gauge redundancies since Eqs. (11), (12) are equations of motion for the expectation value of a gauge invariant operator in the quantum theory. Therefore, the complete observable  $V(\phi)$  can be found by solving the equations of motion. Gauge redundancies can nevertheless be reintroduced at the macroscopic level by means of a time parameter, in order to make contact with the corresponding symmetry of classical cosmology. More specifically, one could write Eq. (13) with the velocity of the field

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<sup>1</sup>In the canonical formalism such space is defined as the space of gauge orbits generated by the first class constraints on the constraint surface [7].

evaluated w.r.t. a time  $t'$  distinct from proper time  $t$  as

$$\pi_\phi = N^{-1}V \frac{d\phi}{dt'}, \quad N = \frac{dt}{dt'}, \quad (30)$$

using which it is possible to write the dynamical equations for all time parametrizations. From the above it is clear how Eq. (15) follows as a consequence of the choice of a specific time parameter, or equivalently lapse function  $N = 1$ .

We have shown that the bounce is accompanied by an early stage of accelerated expansion, occurring for any values of the conserved quantities  $E$ ,  $Q$  (provided that the latter is non-vanishing) and despite the fact that no potential has been introduced for the scalar field. This is a promising feature of the model which indicates that the framework adopted allows for a mechanism leading to an accelerated expansion through quantum geometry effects. From the point of view of Eq. (15) one can say that the accelerated expansion is a consequence of  $G_{eff}$  not being constant. By looking at Eqs. (16), (17), (27) one sees that this phenomenon can be traced back to an effective fluid component having a negative energy density arising from quantum geometry effects. The model we considered is a very simple one and represents a first step towards a new understanding of cosmology in the GFT framework. However, there are some caveats. In fact, a full viability of the scenario of *geometric inflation* can only be proven by showing the robustness of the result when considering more complicated GFT models with different matter fields coupled to gravity. Only then one would be able to give a definite answer as whether the era of accelerated expansion lasts long enough to cure the shortcomings of the standard Hot Big Bang model, while it eventually leads to a radiation-dominated era through a graceful exit scenario. Nevertheless, despite the simplicity of the model, we expect it to provide a good description of the dynamics of the Universe at least at the onset of inflation, where the energy density of the scalar field is supposed to dominate over all other forms of energy.

#### 4. Outlook and conclusions

We studied the properties of a model of quantum cosmology obtained in Ref. [5] in the hydrodynamic limit of GFT. We have shown that this model displays significant departures from the dynamics of a classical FLRW spacetime. The emergent classical background satisfies an evolution equation

of the Friedmann type with quantum corrections appearing in the r.h.s as effective fluids with distinct equations of state. Such correction terms vanish in the limit of infinite volume, where the standard Friedmann dynamics is recovered.

The main results of this work are two. First, confirming the result of Ref. [5], we have shown that there is a *bounce*, taking place regardless of the particular values of the conserved charges  $Q$  and  $E$ . It should be pointed out that the origin of this bounce is quite different from the one given by Loop Quantum Cosmology (see Refs. [10, 11]), which is an independent approach based on a symmetry reduced quantization.

The second result is the occurrence of an era of *accelerated expansion* without the need for introducing *ad hoc* potentials and initial conditions for a scalar field. We suggest that the picture given could replace the inflationary scenario. Since it is an inherently quantum description of cosmology, it does not share the unsatisfactory features of inflationary models, which were spelled out in the introduction. However, the viability of our model as an alternative to inflation is at this stage still an hypothesis, which will be investigated further in future work. In fact the model must be extended to include also other forms of energy and to ensure that common problems of inflationary models (as those spelled out in the last part of Section 3) are solved.

We have seen that the interesting features of the model arise from quantum geometry corrections which are captured by a description in terms of effective fluids defined on the emergent classical background. A similar phenomenon was already observed in LQC (see *e.g.* [6]). In light of our results, it will be interesting to understand whether it is possible to relate the origin of such effective fluids coming from quantum geometry in LQC and GFT. We also showed that there is another way of formulating the dynamics, which makes no reference to such effective fluids, but instead differs from the standard Friedmann equation in that the gravitational constant is replaced by a dynamical quantity. In fact, another interesting result is that, even though Newton's constant is related to, and actually constrains, the parameters of the microscopic GFT theory (as shown in [5]), the dynamics of the expansion of the Universe is actually determined by the *effective gravitational constant*  $G_{\text{eff}}$ . We should stress that such quantity was introduced in first place for the only purpose of studying the properties of solutions of the model. Nevertheless, it is tempting to go one step further and consider it as an effective macroscopic quantity determined by the collective behaviour of spacetime

quanta. However, such an interpretation would possibly pose more puzzles than it solves since, as shown in Refs. [12], [13], a dynamical gravitational constant must bear with it extra sources of energy-momentum in order to ensure compatibility with the Bianchi identities. Violation of the Bianchi identities would in fact imply that the structure of the emergent spacetime is non-Riemannian. The way in which inflation could be understood in that case is not clear and would deserve further study. Nevertheless, the formally equivalent description of the dynamics (as far as the Friedmann equation is concerned) in terms of an effective gravitational constant deserves further investigation. More precisely, if interpretational issues in the non-Riemannian framework can be properly addressed, such studies might shed some light on the nature of the gravitational constant and point out whether it actually deserves the status of fundamental constant, along with the possibility of measuring its time variation.

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